CHAPTER 1

Trigonometric Functions & Equations

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx$ is an identity in x, where $C_0, C_1, ...C_n$ are constants, and $C_n \neq 0$. then the value of n is ________ *(1981 - 2 Marks)*

2. The solution set of the system of equations $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real, is ________. *(1987 - 2 Marks)*

3. The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$, is ________. *(1987 - 2 Marks)*

4. The sides of a triangle inscribed in a given circle subtend angles α, β and γ at the centre. The minimum value of the arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right)$, $\cos\left(\beta + \frac{\pi}{2}\right)$ and $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to ________ *(1987 - 2 Marks)*

5. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$ is equal to ________ *(1991 - 2 Marks)*

6. If $K = \sin(\pi/18)\sin(5\pi/18)\sin(7\pi/18)$, then the numerical value of K is ________. *(1993 - 2 Marks)*

7. If $A > 0, B > 0$ and $A + B = \pi/3$, then the maximum value of $\tan A \tan B$ is________ . *(1993 - 2 Marks)*

8. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is________. *(1996 - 1 Mark)*

9. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are ..., ..., and________. *(1997 - 2 Marks)*

B True / False

1. If $\tan A = (1 - \cos B)/\sin B$, then $\tan 2A = \tan B$. *(1983 - 1 Mark)*

2. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$. *(1984 - 1 Mark)*

C MCQs with One Correct Answer

1. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is *(1979)*
   (a) $-\frac{4}{5}$ but not $\frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$
   (c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) None of these.

2. If $\alpha + \beta + \gamma = 2\pi$, then *(1979)*
   (a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   (b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
   (c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   (d) None of these.

3. Given $A = \sin^2\theta + \cos^4\theta$ then for all real values of θ *(1980)*
   (a) $1 \leq A \leq 2$ (b) $\frac{3}{4} \leq A \leq 1$
   (c) $\frac{13}{16} \leq A \leq 1$ (d) $\frac{3}{4} \leq A \leq \frac{13}{16}$

4. The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}$; $0 < x \leq \frac{\pi}{2}$ has
   (a) no real solution (b) one real solution
   (c) more than one solution (d) none of these *(1980)*

5. The general solution of the trigonometric equation $\sin x + \cos x = 1$ is given by : *(1981 - 2 Marks)*
   (a) $x = 2n\pi$; $n = 0, \pm1, \pm2...$
   (b) $x = 2n\pi + \pi/2$; $n = 0, \pm1, \pm2...$
   (c) $x = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$, $n = 0, \pm1, \pm2...$
   (d) none of these

6. The value of the expression $\sqrt{3}$ cosec 20° – sec 20° is equal to *(1988 - 2 Marks)*
(a) 2 (b) 2 sin 20°/sin 40°
(c) 4 (d) 4 sin 20°/sin 40°

7. The general solution of
$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is *(1989 - 2 Marks)*
(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$
(c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (d) $2n\pi + \cos^{-1}\frac{3}{2}$

8. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$
In the variable x, has real roots. Then p can take any value in the interval *(1990 - 2 Marks)*
(a) $(0, 2\pi)$ (b) $(-\pi, 0)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

9. Number of solutions of the equation *(1993 - 1 Mark)*
$\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is :
(a) 0 (b) 1 (c) 2 (d) 3

10. Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equals *(1994)*
(a) $\tan\left(x - \frac{\pi}{4}\right)$ (b) $\tan\left(\frac{\pi}{4} - x\right)$
(c) $\tan\left(x + \frac{\pi}{4}\right)$ (d) $\tan^2\left(x + \frac{\pi}{4}\right)$

11. Let n be a positive integer such that
$\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$. Then *(1994)*
(a) $6 \le n \le 8$ (b) $4 < n \le 8$
(c) $4 \le n \le 8$ (d) $4 < n < 8$

12. If ω is an imaginary cube root of unity then the value of
$\sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right\}$ is *(1994)*
(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

13. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$ *(1995S)*
(a) 11 (b) 12 (c) 13 (d) 14

14. The general values of θ satisfying the equation
$2\sin^2\theta - 3\sin\theta - 2 = 0$ is *(1995S)*
(a) $n\pi + (-1)^n \pi/6$ (b) $n\pi + (-1)^n \pi/2$
(c) $n\pi + (-1)^n 5\pi/6$ (d) $n\pi + (-1)^n 7\pi/6$

15. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if *(1996 - 1 Mark)*
(a) $x + y \ne 0$ (b) $x = y, x \ne 0$
(c) $x = y$ (d) $x \ne 0, y \ne 0$

16. In a triangle PQR, $\angle R = \pi/2$. If $\tan(P/2)$ and $\tan(Q/2)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \ne 0)$ then. *(1999 - 2 Marks)*
(a) $a + b = c$ (b) $b + c = a$
(c) $a + c = b$ (d) $b = c$

17. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is *(2000S)*
(a) ≥ 0 only when $\theta \ge 0$ (b) ≤ 0 for all real θ
(c) ≥ 0 for all real θ (d) ≤ 0 only when $\theta \le 0$

18. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$ $= 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is *(2001S)*
(a) 0 (b) 2 (c) 1 (d) 3

19. The maximum value of $(\cos\alpha_1).(\cos\alpha_2)\ldots(\cos\alpha_n)$, under the restrictions
$0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2)\ldots(\cot\alpha_n) = 1$ is *(2001S)*
(a) $1/2^{n/2}$ (b) $1/2^n$ (c) $1/2n$ (d) 1

20. If $\alpha + \beta = \pi/2$ and $\beta + \gamma = \alpha$, then $\tan\alpha$ equals *(2001S)*
(a) $2(\tan\beta + \tan\gamma)$ (b) $\tan\beta + \tan\gamma$
(c) $\tan\beta + 2\tan\gamma$ (d) $2\tan\beta + \tan\gamma$

21. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is *(2002S)*
(a) 4 (b) 8 (c) 10 (d) 12

22. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to *(2004S)*
(a) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (d) $\left(\frac{5\pi}{6}, \pi\right]$

23. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$. Pairs of α, β which satisfy both the equations is/are *(2005S)*
(a) 0 (b) 1 (c) 2 (d) 4

24. The values of $\theta \in (0, 2\pi)$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, are *(2006 - 3M, –1)*
(a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (b) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
(c) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{41\pi}{48}, \pi\right)$

25. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then *(2006 - 3M, –1)*
(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$
(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

26. The number of solutions of the pair of equations
$2\sin^2\theta - \cos 2\theta = 0$
$2\cos^2\theta - 3\sin\theta = 0$
in the interval $[0, 2\pi]$ is *(2007 - 3 Marks)*
(a) zero (b) one (c) two (d) four

27. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has *(JEE Adv. 2014)*
(a) infinitely many solutions
(b) three solutions
(c) one solution
(d) no solution

28. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to *(JEE Adv. 2016)*
(a) $-\frac{7\pi}{9}$ (b) $-\frac{2\pi}{9}$
(c) 0 (d) $\frac{5\pi}{9}$

29. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to *(JEE Adv. 2016)*
(a) $3 - \sqrt{3}$ (b) $2(3 - \sqrt{3})$
(c) $2(\sqrt{3} - 1)$ (d) $2(2 - \sqrt{3})$

D MCQs with One or More than One Correct

1. $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$ is equal to *(1984 - 3 Marks)*
(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$
(c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

2. The expression $3\left[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right]$ is equal to *(1986 - 2 Marks)*
(a) 0 (b) 1
(c) 3 (d) $\sin 4\alpha + \cos 6\alpha$
(e) none of these

3. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2\cos(2x) + a_3\sin^2(x) = 0$ for all x is *(1987 - 2 Marks)*
(a) zero (b) one (c) three
(d) infinite (e) none

4. The values of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation *(1988 - 2 Marks)*
$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ are}$$
(a) $7\pi/24$ (b) $5\pi/24$ (c) $11\pi/24$ (d) $\pi/24$.

5. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval *(1994)*
(a) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (b) $\left(-1, \frac{5\pi}{6}\right)$
(c) $(-1, 2)$ (d) $\left(\frac{\pi}{6}, 2\right)$

6. The minimum value of the expression $\sin\alpha + \sin\beta + \sin\gamma$, where α, β, γ are real numbers satisfying $\alpha + \beta + \gamma = \pi$ is *(1995)*
(a) positive (b) zero
(c) negative (d) –3

7. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is *(1998 - 2 Marks)*
(a) 0 (b) 5 (c) 6 (d) 10

8. Which of the following number(s) is/are rational? *(1998 - 2 Marks)*
(a) sin 15° (b) cos 15°
(c) sin 15° cos 15° (d) sin 15° cos 75°

9. For a positive integer n, let *(1999 - 3 Marks)*
$$f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)....(1+\sec 2^n\theta).$$
Then
(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$
(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$

10. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then *(2009)*
(a) $\tan^2 x = \frac{2}{3}$ (b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
(c) $\tan^2 x = \frac{1}{3}$ (d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

11. For $0 < \theta < \frac{\pi}{2}$, the solution (s) of

$$\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$$

is (are) *(2009)*

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{5\pi}{12}$

12. Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\varphi) = \sin^2\theta$ $\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1, \tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$, then φ cannot satisfy *(2012)*

(a) $0 < \varphi < \frac{\pi}{2}$ (b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$

(c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ (d) $\frac{3\pi}{2} < \varphi < 2\pi$

13. The number of points in $(-\infty \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is *(JEE Adv. 2013)*

(a) 6 (b) 4 (c) 2 (d) 0

14. Let $f(x) = x \sin \pi x, x > 0$. Then for all natural numbers n, $f'(x)$ vanishes at *(JEE Adv. 2013)*

(a) A unique point in the interval $\left(n, n+\frac{1}{2}\right)$

(b) A unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

(c) A unique point in the interval $(n, n+1)$

(d) Two points in the interval $(n, n+1)$

E Subjective Problems

1. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, find the possible values of $(\alpha + \beta)$. *(1978)*

2. (a) Draw the graph of $y = \frac{1}{\sqrt{2}}(\sin x + \cos x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.

(b) If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$, and α, β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$. *(1979)*

3. Given $\alpha + \beta - \gamma = \pi$, prove that $\sin^2\alpha + \sin^2\beta - \sin^2\gamma = 2\sin\alpha\sin\beta\cos\gamma$ *(1980)*

4. Given $A = \left\{x : \frac{\pi}{6} \le x \le \frac{\pi}{3}\right\}$ and $f(x) = \cos x - x(1+x)$; find $f(A)$. *(1980)*

5. For all θ in $[0, \pi/2]$ show that, $\cos(\sin\theta) \ge \sin(\cos\theta)$. *(1981 - 4 Marks)*

6. Without using tables, prove that $(\sin 12°)(\sin 48°)(\sin 54°) = \frac{1}{8}$. *(1982 - 2 Marks)*

7. Show that $16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right) = 1$ *(1983 - 2 Marks)*

8. Find all the solution of $4\cos^2 x\sin x - 2\sin^2 x = 3\sin x$ *(1983 - 2 Marks)*

9. Find the values of $x \in (-\pi, +\pi)$ which satisfy the equation $8^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+......)} = 4^3$ *(1984 - 2 Marks)*

10. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$ *(1988 - 2 Marks)*

11. ABC is a triangle such that

$$\sin(2A + B) = \sin(C - A) = -\sin(B + 2C) = \frac{1}{2}.$$

If A, B and C are in arithmetic progression, determine the values of A, B and C. *(1990 - 5 Marks)*

12. If exp $\{(\sin^2 x + \sin^4 x + \sin^6 x + \text{...............} \infty) \ln 2\}$ satisfies the equation $x^2 - 9x + 8 = 0$, find the value of $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$. *(1991 - 4 Marks)*

13. Show that the value of $\frac{\tan x}{\tan 3x}$, wherever defined never lies between $\frac{1}{3}$ and 3. *(1992 - 4 Marks)*

14. Determine the smallest positive value of x (in degrees) for which $\tan(x + 100°) = \tan(x + 50°)\tan(x)\tan(x - 50°)$. *(1993 - 5 Marks)*

15. Find the smallest positive number p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has a solution $x \in [0, 2\pi]$. *(1995 - 5 Marks)*

16. Find all values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$. *(1996 - 2 Marks)*

17. Prove that the values of the function $\frac{\sin x\cos 3x}{\sin 3x\cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x. *(1997 - 5 Marks)*

18. Prove that $\sum_{k=1}^{n-1}(n-k)\cos\frac{2k\pi}{n} = -\frac{n}{2}$, where $n \ge 3$ is an integer. *(1997 - 5 Marks)*

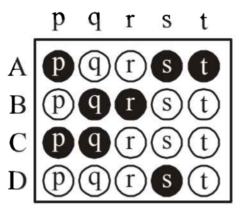
19. In any triangle ABC, prove that *(2000 - 3 Marks)*

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}.$$

20. Find the range of values of t for which $2\sin t = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. *(2005 - 2 Marks)*

F Match the Following

DIRECTIONS (Q. 1): *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. In this questions there are entries in columns 1 and 2. Each entry in column 1 is related to exactly one entry in column 2. Write the correct letter from column 2 against the entry number in column 1 in your answer book.

$\dfrac{\sin 3\alpha}{\cos 2\alpha}$ is *(1992 - 2 Marks)*

Column I	Column II
(A) positive	(p) $\left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$
(B) negative	(q) $\left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$
	(r) $\left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$
	(s) $\left(0, \frac{\pi}{2}\right)$

I Integer Value Correct Type

1. The number of all possible values of θ where $0 < \theta < \pi$, for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is *(2010)*

2. The number of values of θ in the interval, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is *(2010)*

3. The maximum value of the expression

$$\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$$ is *(2010)*

4. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is *(2010)*

[**Note** : $[k]$ denotes the largest integer less than or equal to k]

5. The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$ is *(2011)*

6. The number of distinct solutions of the equation

$$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

in the interval $[0, 2\pi]$ is *(JEE Adv. 2015)*

Section-B JEE Main / AIEEE

1. The period of $\sin^2\theta$ is [2002]
(a) π^2 (b) π (c) 2π (d) $\pi/2$

2. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is [2002]
(a) 2 (b) 3 (c) 0 (d) 1

3. Which one is not periodic [2002]
(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$
(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$

4. Let α, β be such that $\pi < \alpha - \beta < 3\pi$.
If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ [2004]
(a) $\frac{-6}{65}$ (b) $\frac{3}{\sqrt{130}}$
(c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

5. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by [2004]
(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$
(c) $(a+b)^2$ (d) $2(a^2+b^2)$

6. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals [2004]
(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

7. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is [2006]
(a) 4 (b) 6 (c) 1 (d) 2

8. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then tan x is [2006]
(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$
(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

9. Let **A** and **B** denote the statements
A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$
If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then : [2009]
(a) **A** is false and **B** is true (b) both **A** and **B** are true
(c) both **A** and **B** are false (d) **A** is true and **B** is false

10. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ = [2010]
(a) $\frac{56}{33}$ (b) $\frac{19}{12}$ (c) $\frac{20}{7}$ (d) $\frac{25}{16}$

11. If $A = \sin^2 x + \cos^4 x$, then for all real x : [2011]
(a) $\frac{13}{16} \le A \le 1$ (b) $1 \le A \le 2$
(c) $\frac{3}{4} \le A \le \frac{13}{16}$ (d) $\frac{3}{4} \le A \le 1$

12. In a ΔPQR, If $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to : [2012]
(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

13. ABCD is a trapezium such that AB and CD are parallel and BC $\perp$ CD. If $\angle$ADB $= \theta$, BC $= p$ and CD $= q$, then AB is equal to : [JEE M 2013]
(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (b) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$
(c) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ (d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

14. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as : [JEE M 2013]
(a) sinA cosA + 1 (b) secA cosecA + 1
(c) tanA + cotA (d) secA + cosecA

15. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals [JEE M 2014]
(a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

16. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is: [JEE M 2016]
(a) 7 (b) 9
(c) 3 (d) 5

Solutions & Explanations

1 Trigonometric Functions & Equations

Section-A : JEE Advanced/ IIT-JEE

A **1.** 6 **2.** ϕ **3.** $\left[0, \frac{\pi}{6}\right] \cup \left\{\frac{\pi}{2}\right\} \cup \left[\frac{5\pi}{6}, \pi\right]$ **4.** $\frac{-\sqrt{3}}{2}$

5. $\frac{1}{64}$ **6.** $\frac{1}{8}$ **7.** $\frac{1}{3}$ **8.** $n\pi,\ n\pi \pm \frac{\pi}{3}$ **9.** $-\frac{\pi}{2}, \frac{\pi}{2}, 0$

B **1.** T **2.** F

C

1. (b)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (b)	**8.** (d)
9. (c)	**10.** (b)	**11.** (d)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (a)
17. (c)	**18.** (c)	**19.** (a)	**20.** (c)	**21.** (b)	**22.** (b)	**23.** (d)	**24.** (a)
25. (b)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (c)			

D

1. (c)	**2.** (b)	**3.** (d)	**4.** (a, c)	**5.** (d)	**6.** (c)	**7.** (c)	**8.** (c)
9. (a,b,c,d)		**10.** (a, b)	**11.** (c, d)	**12.** (a, c, d)	**13.** (c)	**14.** (b, c)	

E **1.** $n\pi + \frac{\pi}{4}$ **2.** (b) $\frac{56}{33}$ **4.** $\left[\frac{1}{2} - \frac{\pi}{3}\left(1+\frac{\pi}{3}\right), \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1+\frac{\pi}{6}\right)\right]$

8. $n\pi, n\pi + (-1)^n \frac{\pi}{10}, n\pi + (-1)^n \left(\frac{-3\pi}{10}\right)$ **9.** $\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$ **11.** $45°, 60°, 75°$

12. $\frac{\sqrt{3}-1}{2}$ **14.** $30°$ **15.** $\frac{\pi\sqrt{2}}{4}$ **16.** $\pm\frac{\pi}{3}$

20. $\left[-\frac{\pi}{2}, \frac{-\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$

F **1.** (A) - r, (B) - p

I **1.** 3 **2.** 3 **3.** 2 **4.** 3 **5.** 7 **6.** 8

Section-B : JEE Main/ AIEEE

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (c)
9. (b)	**10.** (a)	**11.** (d)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (a)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Given $\sin^3 x . \sin 3x = \sum_{m=0}^{n} C_m \cos mx$

$$\sin^3 x \sin 3x = \frac{1}{4}[3\sin x - \sin 3x]\sin 3x$$

$$= \frac{1}{4}\left[\frac{3}{2}.2\sin x.\sin 3x - \sin^2 3x\right]$$

$$= \frac{1}{4}\left[\frac{3}{2}(\cos 2x - \cos x) - \frac{1}{2}(1-\cos 6x)\right]$$

$$= \frac{1}{8}\left[\cos 6x + 3\cos 2x - 3\cos x - 1\right]$$

We observe that on LHS 6 is the max value of m.

$\therefore \quad n = 6$

2. The equations are $x + y = 2\pi/3$ ... (i)

$\cos x + \cos y = 3/2$ ...(ii)

From eq. (ii) $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{3}{2}$

$\Rightarrow \quad 2\cos\frac{\pi}{3}\cos\frac{x-y}{2} = \frac{3}{2}$ [Using eq. (i)]

$\Rightarrow \quad 2.\frac{1}{2}\cos\frac{x-y}{2} = \frac{3}{2} \Rightarrow \cos\frac{x-y}{2} = \frac{3}{2} > 1$

Which has no solution.

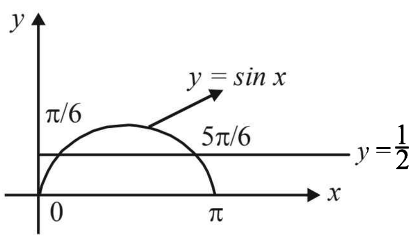
$\therefore$ The solution of given equations is ϕ.

3. We have $2\sin^2 x - 3\sin x + 1 \geq 0$

$\Rightarrow \quad (2\sin x - 1)(\sin x - 1) \geq 0$

$\Rightarrow \quad \left(\sin x - \frac{1}{2}\right)(\sin x - 1) \geq 0 \Rightarrow \sin x \leq \frac{1}{2}$ or $\sin x \geq 1$

But we know that $\sin x \leq 1$ and $\sin x \geq 0$ for $x \in [0, \pi]$



$\Rightarrow \quad$ either $\sin x = 1$ or $0 \leq \sin x \leq \frac{1}{2}$

$\Rightarrow \quad$ either $x = \pi/2$ or $x \in [0, \pi/6] \cup [5\pi/6, \pi]$

Combining, we get $x \in [0, \pi/6] \cup \{\pi/2\} \cup [5\pi/6, \pi]$

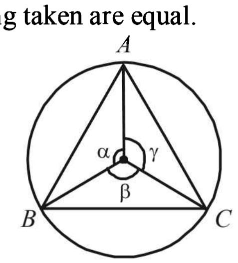
4. We know that A.M. $\geq$ G.M.

$\Rightarrow$ Min value of AM. is obtained when AM = GM

$\Rightarrow$ The quantities whose AM is being taken are equal.

i.e., $\cos\left(\alpha + \frac{\pi}{2}\right) = \cos\left(\beta + \frac{\pi}{2}\right)$

$= \cos\left(\gamma + \frac{\pi}{2}\right)$



$\Rightarrow \quad \sin\alpha = \sin\beta = \sin\gamma$

Also $\alpha + \beta + \gamma = 360° \Rightarrow \alpha = \beta = \gamma = 120° = 2\pi/3$

$\therefore$ Min value of A.M.

$$= \frac{\cos\left(\frac{2\pi}{3} + \frac{\pi}{2}\right) + \cos\left(\frac{2\pi}{3} + \frac{\pi}{2}\right) + \cos\left(\cos\frac{2\pi}{3} + \frac{\pi}{2}\right)}{3}$$

$$= \frac{-\sin\frac{2\pi}{3}}{3} = -\frac{\sqrt{3}}{2}$$

5. $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$

$$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2$$

$$= \left[\cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)\right]^2$$

$$= \left[\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right]^2 = \left[\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right]^2$$

$$= \left(\frac{1}{8\sin\pi/7}\sin\frac{8\pi}{7}\right)^2 = \left(\frac{\sin(\pi + \pi/7)}{8\sin\pi/7}\right)^2$$

$$= \left(\frac{-\sin\pi/7}{8\sin\pi/7}\right)^2 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$$

6. $K = \sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$

$$= \cos\left(\frac{\pi}{2} - \frac{\pi}{18}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{18}\right)\cos\left(\frac{\pi}{2} - \frac{7\pi}{18}\right)$$

$$= \cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{2^3\sin\frac{\pi}{9}}.\sin\frac{8\pi}{9}$$

[Using $\cos\alpha\cos 2\alpha\cos 2^2\alpha \ldots\ldots\ldots \cos 2^{n-1}\alpha$

$= \frac{1}{2^n\sin\alpha}.\sin(2^n\alpha)$]

$$= \frac{1}{8\sin\pi/9}.\sin\pi/9 = \frac{1}{8}$$

7. $A + B = \pi/3 \Rightarrow \tan(A+B) = \sqrt{3}$

$\Rightarrow \quad \frac{\tan A + \tan B}{1 - \tan A\tan B} = \sqrt{3} \Rightarrow \frac{\tan A + \frac{y}{\tan A}}{1-y} = \sqrt{3}$

[where $y = \tan A\tan B$]

$\Rightarrow \tan^2 A + \sqrt{3}(y-1)\tan A + y = 0$

For real value of $\tan A$, $3(y-1)^2 - 4y \geq 0$

$\Rightarrow 3y^2 - 10y + 3 \geq 0 \Rightarrow (y-3)(y - \frac{1}{3}) \geq 0$

$\Rightarrow\ y \le \frac{1}{3}$ or ≥ 3

But $A, B > 0$ and $A + B = \pi/3 \Rightarrow A, B < \pi/3$
$\Rightarrow\ \tan A \tan B < 3$

$\therefore\ y \le \frac{1}{3}$ i.e., max. value of y is 1/3.

8. $\tan^2\theta + \sec 2\theta = 1$

$t^2 + \frac{1+t^2}{1-t^2} = 1$ where $t = \tan\theta$

$\therefore\ t^2(t^2 - 3) = 0\ \ \therefore\ \tan\theta = 0, \pm\sqrt{3}$ etc.

which means $\theta = n\pi$ *and* $\theta = n\pi \pm \pi/3$

9. $\cos^7 x = 1 - \sin^4 x = (1-\sin^2 x)(1+\sin^2 x)$
$= \cos^2 x\,(1 + \sin^2 x)$

$\therefore\ \cos x = 0$ or $x = \pi/2, -\pi/2$
or $\cos^5 x = 1 + \sin^2 x$ or $\cos^5 x - \sin^2 x = 1$
Now maximum value of each $\cos x$ or $\sin x$ is 1.
Hence the above equation will hold when
$\cos x = 1$ and $\sin x = 0$. Both these imply $x = 0$

Hence $x = -\frac{\pi}{2}, \frac{\pi}{2}, 0$

B. True/False

1. $\tan A = \frac{1-\cos B}{\sin B} = \frac{2\sin^2 B/2}{2\sin B/2\cos/2} = \tan B/2$

Hence $\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\tan B/2}{1-\tan^2 B/2} = \tan B$

$\therefore$ Statement is true.

2. Given equation is $\sin^4\theta - 2\sin^2\theta - 1 = 0$
Here, $D = 4 + 4 = 8$

$\therefore\ \sin^2\theta = \frac{2\pm\sqrt{8}}{2} = 1 \pm \sqrt{2}.$

But $\sin^2\theta$ can not be –ve $\quad\therefore\ \sin^2\theta = \sqrt{2} + 1$

But as $-1 \le \sin\theta \le 1\ \ \therefore\ \sin^2\theta \ne \sqrt{2} + 1$

Thus there is no value of θ which satisfy the given equation.
$\therefore$ Statement is false.

C. MCQs with ONE Correct Answer

1. **(b)** $\tan\theta = \frac{-4}{3} \Rightarrow \theta \in$ II quad or IV quad .

$\therefore\ 0 < \sin\theta < 1$ or $-1 < \sin\theta < 0$

$\therefore\ \sin\theta$ may be $\frac{4}{5}$ or $-\frac{4}{5}$

2. **(a)** $\alpha + \beta + \gamma = 2\pi \Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$

$\therefore\ \tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \tan\left(\pi - \frac{\gamma}{2}\right) = -\tan\frac{\gamma}{2}$

$\Rightarrow\ \frac{\tan\alpha/2 + \tan\beta/2}{1 - \tan\alpha/2\tan\beta/2} = -\tan\gamma/2$

$\Rightarrow\ \tan\alpha/2 + \tan\beta/2 + \tan\gamma/2$
$= \tan\alpha/2\ \tan\beta/2\ \tan\gamma/2$

3. **(b)** $A = \sin^2\theta + \cos^4\theta = \sin^2\theta + (1-\sin^2\theta)^2$

$= \sin^4\theta - \sin^2\theta + 1 \Rightarrow A = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

But $0 \le \left(\sin^2\theta - \frac{1}{2}\right)^2 \le \frac{1}{4}$

$\therefore\ \frac{3}{4} \le \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \le 1$ or $\frac{3}{4} \le A \le 1$

4. **(a)** The given equation is

$2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2}$ where $0 < x \le \frac{\pi}{2}$

LHS $= 2\cos^2\frac{x}{2}\sin^2 x = (1+\cos x)\sin^2 x$

$\because\ 1 + \cos x < 2$ and $\sin^2 x \le 1$ for $0 < x \le \frac{\pi}{2}$

$\therefore\ (1 + \cos x)\sin^2 x < 2$

And R.H.S. $= x^2 + \frac{1}{x^2} \ge 2\quad\therefore$ For $0 < x \le \frac{\pi}{2}$,

given equation is not possible for any real value of x.

5. **(c)** $\sin x + \cos x = 1$

$\Rightarrow\ \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos = \frac{1}{\sqrt{2}}$

$\Rightarrow\ \sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4} = \sin\frac{\pi}{4}$

$\Rightarrow\ \sin(x + \pi/4) = \sin\pi/4$
$\Rightarrow\ x + \pi/4 = n\pi + (-1)^n\pi/4, n \in Z$ (the set of integers)
$\Rightarrow\ x = n\pi + (-1)^n\pi/4 - \pi/4$
where n = 0, ± 1, ± 2, ...

6. **(c)** The given expression is $\sqrt{3}\,\mathrm{cosec}\,20° - \sec 20°$

$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°\cos 20°}$

$$=4\left[\frac{\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°}{2\sin 20°\cos 20°}\right]$$

$$=4\left[\frac{\sin 60°\cos 20° - \cos 60°\sin 20°}{\sin(2\times 20°)}\right]$$

$$=\frac{4\sin(60°-20°)}{\sin 40°}=\frac{4\sin 40°}{\sin 40°}=4$$

7. (b) The given equation is

$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow 2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cos x - 3\cos 2x$

$\Rightarrow \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$

$\Rightarrow \sin 2x = \cos 2x$ (as $\cos x \neq 3/2$)

$\Rightarrow \tan 2x = 1 \Rightarrow 2x = n\pi + \pi/4 \Rightarrow x = \frac{n\pi}{2}+\frac{\pi}{8}$

8. (d) The given equation is

$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$

For this equation to have real roots $D \geq 0$

$\Rightarrow \cos^2 p - 4\sin p(\cos p - 1) \geq 0$

$\Rightarrow \cos^2 p - 4\sin p\cos p + 4\sin^2 p + 4\sin p - 4\sin^2 p \geq 0$

$\Rightarrow (\cos p - 2\sin p)^2 + 4\sin p(1-\sin p) \geq 0$

For every real value of p $(\cos p - 2\sin p)^2 \geq 0$ and $1-\sin p \geq 0$ $\therefore D \geq 0, \forall p \in (0,\pi)$

9. (c) The given eq is, $\tan x + \sec x = 2\cos x$

$\Rightarrow \frac{\sin x}{\cos x}+\frac{1}{\cos x}=2\cos x$

$\Rightarrow \sin x + 1 = 2\cos^2 x \Rightarrow 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = \frac{1}{2}, -1$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \in [0, 2\pi]$

But for $x = \frac{3\pi}{2}$, given eq. is not defined,

$\therefore$ only 2 solutions.

10. (b) $\sec 2x - \tan 2x = \frac{1-\sin 2x}{\cos 2x} = \frac{1-\cos 2\left(\frac{\pi}{4}-x\right)}{\sin 2\left(\frac{\pi}{4}-x\right)}$

$$=\frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)}=\tan\left(\frac{\pi}{4}-x\right)$$

11. (d) $\sin\frac{\pi}{2n}+\cos\frac{\pi}{2n}=\frac{\sqrt{n}}{2}$

$\Rightarrow \sin^2\frac{\pi}{2n}+\cos^2\frac{\pi}{2n}+2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}=\frac{n}{4}$

$\Rightarrow 1+\sin\frac{\pi}{n}=\frac{n}{4}\Rightarrow \sin\frac{\pi}{n}=\frac{n-4}{4}$

For $n = 2$ the given equation is not satisfied.

Considering $n > 1$ and $n \neq 2$

$0 < \sin\frac{\pi}{n} < 1 \Rightarrow 0 < \frac{n-4}{4} < 1 \Rightarrow 4 < n < 8.$

12. (c) $\sin\left\{(\omega^{10}+\omega^{23})\pi - \frac{\pi}{4}\right\} = \sin\left\{(\omega+\omega^2)\pi - \frac{\pi}{4}\right\}$

$= \sin\left(-\pi-\frac{\pi}{4}\right) = -\sin\left(\pi+\frac{\pi}{4}\right) = \sin\pi/4 = 1/\sqrt{2}$

13. (c) $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

$= 3(1-\sin 2x)^2 + 6(1+\sin 2x)$

$+ 4[(\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)]$

$= 3 - 6\sin 2x + 3\sin^2 2x + 6 + 6\sin 2x + 4\left[1-\frac{3}{4}\sin^2 2x\right]$

$= 13 + 3\sin^2 2x - 3\sin^2 2x = 13$

14. (d) The given equation is $2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$ [$\because \sin\theta - 2 = 0$ is not possible]

$\Rightarrow \sin\theta = \sin(-\pi/6) = \sin(7\pi/6)$

$\Rightarrow \theta = n\pi + (-1)^n(-\pi/6) = n\pi + (-1)^n 7\pi/6$

$\Rightarrow$ Thus, $\theta = n\pi + (-1)^n 7\pi/6$

15. (b) We have $\sec^2\theta = \frac{4xy}{(x+y)^2}$

But $\sec^2\theta \geq 1 \Rightarrow \frac{4xy}{(x+y)^2} \geq 1$

$\Rightarrow 4xy \geq x^2 + y^2 + 2xy$

$\Rightarrow x^2 + y^2 - 2xy \leq 0 \Rightarrow (x-y)^2 \leq 0$

$\Rightarrow x - y = 0$ [as perfect square of real number can never be negative]

Also then $x \neq 0$ as then $\sec^2\theta$ will become indeterminate.

16. (a) Given that in ΔPQR, $\angle R = \pi/2$

$\Rightarrow \angle P + \angle Q = \pi/2 \Rightarrow \frac{\angle P}{2}+\frac{\angle Q}{2}=\frac{\pi}{4}$

Also tan $P/2$ and tan $Q/2$ are roots of the equation $ax^2 + bx + c = 0\ (a \neq 0)$

$\therefore \tan P/2 + \tan Q/2 = -\frac{b}{a}; \tan P/2 \tan Q/2 = c/a$

Now consider, $\tan\left(\frac{P+Q}{2}\right) = \frac{\tan P/2 + \tan Q/2}{1 - \tan P/2 \tan Q/2}$

$\Rightarrow \tan\frac{\pi}{4} = \frac{-b/a}{1-c/a} \Rightarrow 1 - \frac{c}{a} = -\frac{b}{a}$

$\Rightarrow a - c = -b \Rightarrow a + b = c$

17. (c) $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$= (\sin\theta + 3\sin\theta - 4\sin^3\theta).\sin\theta$

$= (4\sin\theta - 4\sin^3\theta)\sin\theta = \sin^2\theta(4 - 4\sin^2\theta)$

$= 4\sin^2\theta\ (1 - \sin^2\theta)$

$= 4\sin^2\theta\cos^2\theta = (2\sin\theta\cos\theta)^2 = (\sin 2\theta)^2 \geq 0$

which is true for all θ.

18. (c) To simplify the det. Let $\sin x = a$; $\cos x = b$ the equation becomes

$\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} = 0$ Operating $C_2 - C_1$; $C_3 - C_2$ we get

$\begin{vmatrix} a & b-a & 0 \\ b & a-b & b-a \\ b & 0 & a-b \end{vmatrix} = 0$

$\Rightarrow a(a-b)^2 - (b-a)[b(a-b) - b(b-a)] = 0$

$\Rightarrow a(a-b)^2 - 2b(b-a)(a-b) = 0$

$\Rightarrow (a-b)^2(a-2b) = 0 \Rightarrow (a = b)$ or $a = 2b$

$\Rightarrow \frac{a}{b} = 1$ or $\frac{a}{b} = 2$

$\Rightarrow \tan x = 1$ or $\tan x = 2$. But we have $-\pi/4 \leq x \leq \pi/4$

$\Rightarrow \tan(-\pi/4) \leq \tan x \leq \tan(\pi/4) \Rightarrow -1 \leq \tan x \leq 1$

$\therefore \tan x = 1 \Rightarrow x = \pi/4$ $\therefore$ Only one real root is there.

19. (a) We are given that

$(\cot\alpha_1).(\cot\alpha_2)\ ....\ (\cot\alpha_n) = 1$

$\Rightarrow (\cos\alpha_1)(\cos\alpha_2)\ ....\ (\cos\alpha_n)$

$= (\sin\alpha_1)(\sin\alpha_2)....(\sin\alpha_n)$ ....(1)

Let $y = (\cos\alpha_1)(\cos\alpha_2)\ ....\ (\cos\alpha_n)$ (to be max.)

Squaring both sides, we get

$y^2 = (\cos^2\alpha_1)(\cos^2\alpha_2)\ ....\ (\cos^2\alpha_n)$

$= \cos\alpha_1 \sin\alpha_1 \cos\alpha_2 \sin\alpha_2\ ....\ \cos\alpha_n \sin\alpha_n$ (Using (1))

$= \frac{1}{2^n}[\sin 2\alpha_1 \sin 2\alpha_2\ ....\ \sin 2\alpha_n]$

As $0 \leq \alpha_1, \alpha_2, ...\alpha_n \leq \pi/2$

$\therefore 0 \leq 2\alpha_1, 2\alpha_2, ...2\alpha_n \leq \pi$

$\Rightarrow 0 \leq \sin 2\alpha_1, \sin 2\alpha_2, ...\sin 2\alpha_n \leq 1$

$\therefore y^2 \leq \frac{1}{2^n}.1 \Rightarrow y \leq \frac{1}{2^{n/2}}$

$\therefore$ Max. value of y is $1/2^{n/2}$.

20. (c) Given that $\alpha + \beta = \pi/2 \Rightarrow \alpha = \pi/2 - \beta$

$\Rightarrow \tan\alpha = \tan(\pi/2 - \beta) = \cot\beta = \frac{1}{\tan\beta}$

$\Rightarrow \tan\alpha\tan\beta = 1 \Rightarrow 1 + \tan\alpha\tan\beta = 2.$

$\therefore \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} \Rightarrow \tan\gamma = \frac{\tan\alpha - \tan\beta}{2}$

$\Rightarrow 2\tan\gamma = \tan\alpha - \tan\beta \Rightarrow \tan\alpha = 2\tan\gamma + \tan\beta$

21. (b) We know that

$\Rightarrow -\sqrt{a^2+b^2} \leq a\cos\theta + b\sin\theta \leq \sqrt{a^2+b^2}$

NOTE THIS STEP

$\Rightarrow -\sqrt{74} \leq 7\cos x + 5\sin x \leq \sqrt{74}$

$\Rightarrow -\sqrt{74} \leq 2k + 1 \leq \sqrt{74} \Rightarrow -8.6 \leq 2k + 1 \leq 8.6$

$\Rightarrow -4.8 \leq k \leq 3.8$

(considering only integral values)

$\Rightarrow k$ can take 8 integral values.

22. (b) Given that $\sin\theta = 1/2$ and $\cos\phi = 1/3$ and θ and ϕ both are acute angles

$\therefore \theta = \pi/6$ and $0 < \frac{1}{3} < \frac{1}{2}$

or $\cos\pi/2 < \cos\phi < \cos\pi/3$ or $\pi/3 < \phi < \pi/2$

$\therefore \frac{\pi}{3} + \frac{\pi}{6} < \theta + \phi < \frac{\pi}{2} + \frac{\pi}{6}$ or $\frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$

$\Rightarrow \theta + \phi \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

23. (d) Given that $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$

Now, $\cos(\alpha - \beta) = 1 \Rightarrow \alpha - \beta = 0 \Rightarrow \alpha = \beta$

Now, $\cos(\alpha + \beta) = 1/e \Rightarrow \cos 2\alpha = 1/e$

$\because 0 < 1/e < 1$ and $2\alpha \in [-2\pi, 2\pi]$

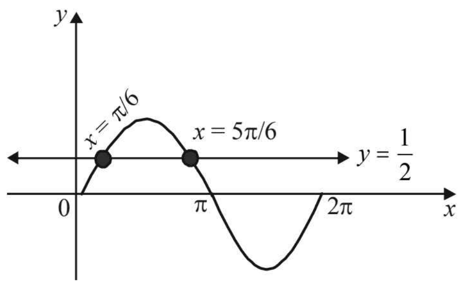
$\Rightarrow$ There will be two values of 2α satisfying $\cos 2\alpha = 1/e$ in $[0, 2\pi]$ and two in $[-2\pi, 0]$.

$\Rightarrow$ There will be four values of α in $[-\pi, \pi]$ and correspondingly four values of β. Hence there are four sets of (α, β).

24. **(a)** $2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow (\sin\theta - 2)(2\sin\theta - 1) > 0$

$\Rightarrow \sin\theta < \frac{1}{2}$ $\quad [\because -1 \le \sin\theta \le 1]$



From graph, we get $x \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

25. **(b)** $\because \theta \in \left(0, \frac{\pi}{4}\right) \Rightarrow \tan\theta < 1$ and $\cot\theta > 1$

Let $\tan\theta = 1 - x$ and $\cot\theta = 1 + y$

Where $x, y > 0$ and are very small, then

$\therefore t_1 = (1-x)^{1-x}, t_2 = (1-x)^{1+y}, t_3 = (1+y)^{1-x}, t_4 = (1+y)^{1+y}$

Clearly, $t_4 > t_3$ and $t_1 > t_2$ also, $t_3 > t_1$ **NOTE THIS STEP**

Thus $t_4 > t_3 > t_1 > t_2$.

26. **(c)** $2\sin^2\theta - \cos 2\theta = 0 \Rightarrow 1 - 2\cos 2\theta = 0$

$\Rightarrow \cos 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ ....(1)

where $\theta \in [0, 2\pi]$

Also $2\cos^2\theta - 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta + 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta + 2) = 0 \Rightarrow \sin\theta = \frac{1}{2}$

$[\because \sin\theta \neq -2]$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ .... (2), where $\theta \in [0, 2\pi]$

Combining (1) and (2), we get $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore$ Two solutions are there.

27. **(d)** $\sin x + 2\sin 2x - \sin 3x = 3$

$\Rightarrow \sin x + 4\sin x\cos x - 3\sin x + 4\sin^3 x = 3$

$\Rightarrow \sin x(-2 + 2\cos x + 4\sin^2 x) = 3$

$\Rightarrow \sin x(-2 + 2\cos x + 4 - 4\cos^2 x) = 3$

$\Rightarrow 2 + 2\cos x - 4\cos^2 x = \frac{3}{\sin x}$

$\Rightarrow 2 - \left(4\cos^2 x - 2 \cdot 2\cos x \cdot \frac{1}{2} + \frac{1}{4}\right) + \frac{1}{4} = \frac{3}{\sin x}$

$\Rightarrow \frac{9}{4} - \left(2\cos x - \frac{1}{2}\right)^2 = \frac{3}{\sin x}$

LHS $\le \frac{9}{4}$ and RHS ≥ 3

$\therefore$ Equation has no solution.

28. **(c)** $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$

$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \cos^2 x - \sin^2 x$

$\Rightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos 2x \Rightarrow x - \frac{\pi}{3} = 2n\pi \pm 2x$

$\Rightarrow x = \frac{2n\pi}{3} + \frac{\pi}{9}$ or $x = -2n\pi - \frac{\pi}{3}$

For $x \in S, n = 0 \Rightarrow x = \frac{\pi}{9}, -\frac{\pi}{3}$

$n = 1 \Rightarrow x = \frac{7\pi}{9}$; $\quad n = -1 \Rightarrow x = \frac{-5\pi}{9}$

$\therefore$ Sum of all values of $x = \frac{\pi}{9} - \frac{\pi}{3} + \frac{7\pi}{9} - \frac{5\pi}{9} = 0$

29. **(c)** $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$

$= \sum_{k=1}^{13} \frac{1}{\sin\frac{\pi}{6}} \left[\frac{\sin\left\{\frac{\pi}{4} + \frac{k\pi}{6} - \left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\right\}}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}\right]$

$\sum_{k=1}^{13} 2\left[\cot\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)\right]$

$= 2\left[\left\{\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right)\right\} + \left\{\cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{2\pi}{6}\right)\right\}\right.$

$\left. + \ldots\ldots + \left\{\cot\left(\frac{\pi}{4} + \frac{12\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)\right\}\right]$

$= 2\left[\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)\right] = 2\left[1 - \cot\frac{5\pi}{12}\right]$

$= 2\left[1 - \frac{\sqrt{3}-1}{\sqrt{3}+1}\right] = 2\left[1 - \left(2 - \sqrt{3}\right)\right] = 2\left(\sqrt{3} - 1\right)$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(c)** We have,

$(1+\cos\pi/8)\ (1+\cos 3\pi/8)(1+\cos 5\pi/8)\ (1+\cos 7\pi/8)$

$= (1+\cos\pi/8)\ (1+\cos 3\pi/8)(1+\cos(\pi-3\pi/8))\ (1+\cos(\pi-\pi/8))$

$= (1+\cos\pi/8)\ (1+\cos 3\pi/8)(1-\cos 3\pi/8)\ (1-\cos\pi/8)$

$= (1-\cos^2\pi/8)\ (1-\cos^2 3\pi/8) = \sin^2\pi/8\ \sin^2 3\pi/8$

$= \frac{1}{4}.[2\sin\pi/8\ \sin(\pi/2-\pi/8)]^2$

$= \frac{1}{4}.[2\sin\pi/8\ \cos\pi/8)]^2 = \frac{1}{4}.\sin^2\pi/4 = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$

∴ (c) is the correct answer.

2. **(b)** The given expression is

$$= 3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]$$
$$-2\,[\sin^6(\pi/2+\alpha)+\sin^6(5\pi-\alpha)]$$

$$= 3\left[\cos^4\alpha+\sin^4\alpha\right]-2\left[\cos^6\alpha+\sin^6\alpha\right]$$

$$= 3\left[\left(\cos^2\alpha+\sin^2\alpha\right)^2-2\sin^2\alpha\cos^2\alpha\right]$$
$$-2\left[(\cos^2\alpha+\sin^2\alpha)^3-3\cos^2\alpha\sin^2\alpha\ (\cos^2\alpha+\sin^2\alpha)\right]$$

$$= 3\left[1-2\sin^2\alpha\cos^2\alpha\right]-2\left[1-3\cos^2\alpha\sin^2\alpha\right]$$

$$= 3-6\sin^2\alpha\cos^2\alpha-2+6\sin^2\alpha\cos^2\alpha = 1$$

3. **(d)** Since $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for all x

Putting $x = 0$ and $x = \pi/2$, we get $a_1 + a_2 = 0$ ....(1)

and $a_1 - a_2 + a_3 = 0$ ....(2)

$\Rightarrow\ a_2 = -a_1$ and $a_3 = -2a_1$

∴ The given equation becomes

$a_1 - a_1\cos 2x - 2a_1\sin^2 x = 0,\ \forall x$

$\Rightarrow\ a_1(1-\cos 2x - 2\sin^2 x) = 0, \forall x$

$\Rightarrow\ a_1(2\sin^2 x - 2\sin^2 x) = 0,\ \forall x$

The above is satisfied for all values of a_1.

Hence infinite number of triplets $(a_1, -a_1, -2a_1)$ are possible.

4. **(a,c)** We have

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Operating $C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Operating $R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Expanding along R_1 we get $[1+4\sin 4\theta+1] = 0$

$\Rightarrow\ 2(1+2\sin 4\theta) = 0 \Rightarrow\ \sin 4\theta = -\frac{1}{2}$

$\Rightarrow\ 4\theta = \pi+\pi/6$ or $2\pi-\pi/6$

$\Rightarrow\ 4\theta = 7\pi/6$ or $11\pi/6$

$\Rightarrow\ \theta = 7\pi/24$ or $11\pi/24$

5. **(d)** $2\sin^2 x + 3\sin x - 2 > 0$

$(2\sin x - 1)(\sin x + 2) > 0$

$\Rightarrow\ 2\sin x - 1 > 0\quad (\because -1 \le \sin x \le 1)$

$\Rightarrow\ \sin x > 1/2 \Rightarrow x \in (\pi/6, 5\pi/6)$ ....(1)

Also $x^2 - x - 2 < 0$

$\Rightarrow\ (x-2)\ (x+1) < 0 \Rightarrow -1 < x < 2$ ....(2)

Combining (1) and (2) $x \in (\pi/6, 2)$.

6. **(c)** $\sin\alpha + \sin\beta + \sin\gamma$

$$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}$$

$$= 2\sin\left(\frac{\pi}{2}-\frac{\gamma}{2}\right)\cos\frac{\alpha-\beta}{2} + 2\sin\left(\frac{\pi}{2}-\frac{\alpha+\beta}{2}\right)\cos\frac{\gamma}{2}$$

$$= 2\cos\frac{\gamma}{2}\left[\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}\right]$$

$= 2\cos\alpha/2\ \cos\beta/2\ \cos\gamma/2$

∴ Each $\cos\alpha/2$, $\cos\beta/2$, $\cos\gamma/2$ lies between -1 and 1, therefore $-1 \le \cos\alpha/2, \cos\beta/2, \cos\gamma/2 \le 1$

$\Rightarrow\ -2 \le 2\cos\alpha/2, \cos\beta/2, \cos\gamma/2 \le 2$

$\Rightarrow\ -2 \le \cos\alpha + \cos\beta + \cos\gamma \le 2$

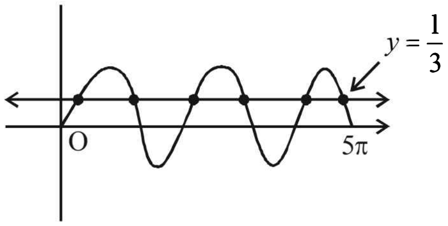
∴ min value $= -2$.

7. **(c)** $3\sin^2 x - 7\sin x + 2 = 0$, put $\sin x = s$

$\Rightarrow\ (s-2)\ (3s-1) = 0\ \Rightarrow\ s = 1/3$

($s = 2$ is not possible)

Number of solutions of $\sin x = \frac{1}{3}$ from the following graph is 6 between $[0, 5\pi]$



8. **(c)** We know that $\sin 15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ (irrational)

$\cos 15° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ (irrational)

$\sin 15° \,.\, \cos 15° = \dfrac{1}{2}\,(2\sin 15°\ \cos 15°)$

$= \dfrac{1}{2}\sin 30° = \dfrac{1}{4}$ (rational)

$\sin 15°\ \cos 75° = \sin 15° \cos(90 - 15°)$

$= \sin 15°\ \sin 15° = \sin^2 15° = \dfrac{1}{2}(1+\cos 30°)$

$= \dfrac{1}{2}\left(1 - \dfrac{\sqrt{3}}{2}\right)$ (irrational)

9. **(a, b, c, d)** $E = f_n(\theta) = \dfrac{\sin(\theta/2)}{\cos(\theta/2)}$

$$\left[\frac{2\cos^2(\theta/2)}{\cos\theta}\cdot\frac{2\cos^2\theta}{\cos 2\theta}\cdot\frac{2\cos^2 2\theta}{\cos 4\theta}\cdots\cdots\frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}\right]$$

$$= \frac{\sin\theta}{\cos\theta}\left[\frac{2\cos^2\theta}{\cos 2\theta}\cdot\frac{2\cos^2 2\theta}{\cos 4\theta}\cdots\cdot\frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}\right] = \tan 2^n\theta\,.$$

$n = 2, \theta = \dfrac{\pi}{16}$

$f_2\left(\dfrac{\pi}{16}\right) = \tan 4.\dfrac{\pi}{16} = \tan\dfrac{\pi}{4} = 1.$

Similarly, $f_3\left(\dfrac{\pi}{32}\right), f_4\left(\dfrac{\pi}{64}\right)$ *and* $f_5\left(\dfrac{\pi}{128}\right)$ *is* $\tan\dfrac{\pi}{4} = 1$.

10. **(a, b)** Given that

$\dfrac{\sin^4 x}{2} + \dfrac{\cos^4 x}{3} = \dfrac{1}{5} \Rightarrow 3\sin^4 x + 2\cos^4 x = \dfrac{6}{5}$

$\Rightarrow \sin^4 x + 2[\sin^4 x + \cos^4 x] = \dfrac{6}{5}$

$\Rightarrow \sin^4 x + 2[1 - 2\sin^2 x\ \cos^2 x] = \dfrac{6}{5}$

$\Rightarrow \sin^4 x + 2 - 4\sin^2 x(1-\sin^2 x) = \dfrac{6}{5}$

$\Rightarrow 5\sin^4 x - 4\sin^2 x + 2 - \dfrac{6}{5} = 0$

$\Rightarrow 25\sin^4 x - 20\sin^2 x + 4 = 0$

$\Rightarrow (5\sin^2 x - 2)^2 = 0 \Rightarrow \sin^2 x = \dfrac{2}{5}$

$\Rightarrow \cos^2 x = \dfrac{3}{5}$ and $\tan^2 x = \dfrac{2}{3}$

Also $\dfrac{\sin^8 x}{8} + \dfrac{\cos^8 x}{27} = \dfrac{2}{625} + \dfrac{3}{625} = \dfrac{5}{625} = \dfrac{1}{125}$

11. **(c, d)** We have

$$\sum_{m=1}^{6} \operatorname{cosec}\left[\theta + \frac{(m-1)\pi}{4}\right]\operatorname{cosec}\left[\theta + \frac{m\pi}{4}\right] = 4\sqrt{2}$$

$$\Rightarrow \sum_{m=1}^{6} \frac{\sin\dfrac{\pi}{4}}{\sin\left[\theta + \dfrac{(m-1)\pi}{4}\right]\sin\left[\theta + \dfrac{m\pi}{4}\right]} = 4$$

$$\Rightarrow \sum_{m=1}^{6} \frac{\sin\left[\left(\theta + \dfrac{m\pi}{4}\right) - \left(\theta + \dfrac{(m-1)\pi}{4}\right)\right]}{\sin\left(\theta + \dfrac{(m-1)\pi}{4}\right)\sin\left(\theta + \dfrac{m\pi}{4}\right)} = 4$$

$$\Rightarrow \sum_{m=1}^{6} \frac{\left[\begin{array}{l}\sin\left(\theta + \dfrac{m\pi}{4}\right)\cos\left(\theta + \dfrac{(m-1)\pi}{4}\right) \\ -\cos\left(\theta + \dfrac{m\pi}{4}\right)\sin\left(\theta + \dfrac{(m-1)\pi}{4}\right)\end{array}\right]}{\sin\left(\theta + \dfrac{(m-1)\pi}{4}\right)\sin\left(\theta + \dfrac{m\pi}{4}\right)} = 4$$

$$\Rightarrow \sum_{m=1}^{6}\left[\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right] = 4$$

$$\Rightarrow \left[\cot\theta - \cot\left(\theta + \frac{\pi}{4}\right)\right] + \left[\cot\left(\theta + \frac{\pi}{4}\right) - \cot\left(\theta + \frac{2\pi}{4}\right)\right]$$

$$+ \ldots + \left[\cot\left(\theta + \frac{5\pi}{4}\right) - \cot\left(\theta + \frac{6\pi}{4}\right)\right] = 4$$

$\Rightarrow \cot\theta - \cot\left(\theta + \dfrac{3\pi}{2}\right) = 4 \Rightarrow \cot\theta + \tan\theta = 4$

$\Rightarrow \cos^2\theta + \sin^2\theta = 4\sin\theta\cos\theta$

$\Rightarrow \sin 2\theta = \dfrac{1}{2} \Rightarrow 2\theta = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6} \Rightarrow \theta = \dfrac{\pi}{12}$ or $\dfrac{5\pi}{12}$

12. **(a, c, d)**

As $\tan(2\pi - \theta) > 0 \Rightarrow -\tan\theta > 0 \Rightarrow \tan\theta < 0$

$\therefore\quad \theta \in$ II or IV quadrant ...(1)

And $-1 < \sin\theta < \dfrac{-\sqrt{3}}{2}$

$\Rightarrow \theta \in$ III or IV quadrant ...(2)

Also $\theta \in [0, 2\pi]$ ...(3)

Combining above three equations (1), (2) and (3); $\theta \in$ IV quadrant and more precisely $\dfrac{3\pi}{2} < \theta < \dfrac{5\pi}{3}$

$\left(\because -1 < \sin\theta < -\dfrac{\sqrt{3}}{2}\right)$

Now,

$2\cos\theta(1 - \sin\phi) = \sin^2\theta(\tan\theta/2 + \cot\theta/2)\cos\phi - 1$

$\Rightarrow 2\cos\theta(1 - \sin\phi) = \sin^2\theta \times \dfrac{1}{\sin\theta/2\ \cos\theta/2}.\cos\phi - 1$

$\Rightarrow 2\cos\theta(1 - \sin\phi) = 2\sin\theta\cos\phi - 1$

$\Rightarrow 2\cos\theta + 1 = 2\sin(\theta + \phi)$

$\because\quad \theta \in \left(\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right)$ $\quad\therefore\ 2\cos\theta + 1 \in (1, 2)$

$\Rightarrow\quad 1 < 2\sin(\theta + \phi) < 2$

$\Rightarrow \dfrac{1}{2} < \sin(\theta + \phi) < 1 \Rightarrow \theta + \phi \in \left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right)$

or $\theta + \phi \in \left(2\pi + \dfrac{\pi}{6}, 2\pi + \dfrac{5\pi}{6}\right)$ or $\left(\dfrac{13\pi}{6}, \dfrac{17\pi}{6}\right)$

But $\theta \in \left(\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right)$ (as $\theta + \phi \in [0, 4\pi]$)

$\Rightarrow \dfrac{13\pi}{6} < \theta + \phi < \dfrac{17\pi}{6} \Rightarrow \dfrac{13\pi}{6} - \theta < \phi < \dfrac{17\pi}{6} - \theta$

$\Rightarrow \dfrac{13\pi}{6} - \theta_{max} < \phi < \dfrac{17\pi}{6} - \theta_{min}$

$\Rightarrow \dfrac{13\pi}{6} - \dfrac{5\pi}{3} < \phi < \dfrac{17\pi}{6} - \dfrac{3\pi}{2} \Rightarrow \dfrac{\pi}{2} < \phi < \dfrac{4\pi}{3}$

13. **(c)** Let $f(x) = x^2 - x\sin x - \cos x$

$\therefore\ f'(x) = 2x - x\cos x = x(2 - \cos x)$

$\therefore\ f$ is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$

Also $\lim\limits_{x\to\infty} f(x) = \infty$, $\lim\limits_{x\to-\infty} f(x) = \infty$ and $f(0) = -1$

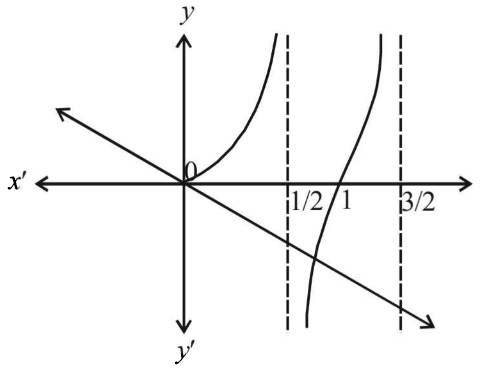
$\therefore\ y = f(x)$ meets x-axis twice.

i.e., $f(x) = 0$ has two points in $(-\infty, \infty)$.

14. **(b, c)** We have $f(x) = x\sin\pi x,\ x > 0$

$\Rightarrow f'(x) = \sin\pi x + x\pi\cos\pi x$

$f'(x) = 0 \Rightarrow \tan\pi x = -\pi x$



We observe, from graph of $y = \tan\pi x$ and $y = -\pi x$ that they intersect at unique point in the intervals

$(n, n+1)$ and $\left(n + \dfrac{1}{2}, n+1\right)$

E. Subjective Problems

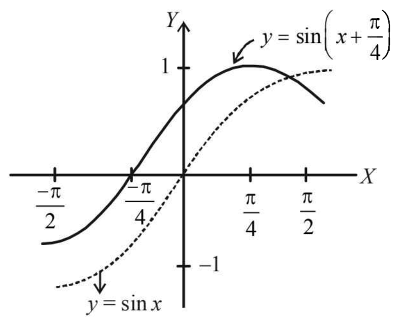
1. We know $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\ \tan\beta}$

$\Rightarrow \tan(\alpha + \beta) = \dfrac{\dfrac{m}{m+1} + \dfrac{1}{2m+1}}{1 - \dfrac{m}{m+1}.\dfrac{1}{2m+1}} = \dfrac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1$

$\Rightarrow \alpha + \beta = n\pi + \pi/4$ where $n \in Z$.

2. **(a)** Given: $y = \dfrac{1}{\sqrt{2}}(\sin x + \cos x) = \sin\left(x + \dfrac{\pi}{4}\right)$ ...(1)

Now, to draw the graph of $y = \sin\left(x + \dfrac{\pi}{4}\right)$, we first draw the graph of $y = \sin x$ and then on shifting it by $-\dfrac{\pi}{4}$, we will obtain the required graph as shown in figure given below.



(b) $\cos(\alpha + \beta) = \dfrac{4}{5}$

$\Rightarrow \tan(\alpha + \beta) = \dfrac{3}{4},\ 0 < \alpha, \beta < \dfrac{\pi}{4}$

$\sin(\alpha-\beta)=\frac{5}{13} \Rightarrow \tan(\alpha-\beta)=\frac{5}{12}$

$\therefore \tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)]$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}=\frac{9+5}{12}\times\frac{16}{11}=\frac{56}{33}$$

3. Given $\alpha+\beta-\gamma=\pi$ and to prove that

$\sin^2\alpha+\sin^2\beta-\sin^2\gamma=2\sin\alpha\sin\beta\cos\gamma$

L.H.S. $=\sin^2\alpha+\sin^2\beta-\sin^2\gamma$

[Using $\sin^2 A-\sin^2 B=\sin(A+B)\sin(A-B)$]

$=\sin^2\alpha+\sin(\beta+\gamma)\sin(\beta-\gamma)$

$=\sin^2\alpha+\sin(\beta+\gamma)\sin(\pi-\alpha)$ $\quad(\because \alpha+\beta-\gamma=\pi)$

$=\sin^2\alpha+\sin(\beta+\gamma)\sin\alpha$

$=\sin\alpha(\sin\alpha+\sin(\beta+\gamma))$

$=\sin\alpha[\sin[\pi-(\beta-\gamma)]+\sin(\beta+\gamma)]$

$=\sin\alpha[\sin(\beta-\gamma)+\sin(\beta+\gamma)]$

$=\sin\alpha[2\sin\beta\cos\gamma]=2\sin\alpha\sin\beta\cos\gamma=$ R.H.S.

4. $A=\left\{x:\frac{\pi}{6}\le x\le\frac{\pi}{3}\right\}$

$f(x)=\cos x-x(1+x)$

$f'(x)=-\sin x-1-2x<0,\ \forall x\in A$

$\therefore f$ is a decreasing function.

$\therefore$ as $\frac{\pi}{6}\le x\le\frac{\pi}{3}\Rightarrow f\left(\frac{\pi}{3}\right)\le f(x)\le\left(\frac{\pi}{6}\right)$

$\Rightarrow \cos\frac{\pi}{3}-\frac{\pi}{3}\left(1+\frac{\pi}{3}\right)\le f(x)\le\cos\frac{\pi}{6}-\frac{\pi}{6}\left(1+\frac{\pi}{6}\right)$

$\therefore f(A)=\left[\frac{1}{2}-\frac{\pi}{3}\left(1+\frac{\pi}{3}\right),\frac{\sqrt{3}}{2}-\frac{\pi}{6}\left(1+\frac{\pi}{6}\right)\right]$

5. We have

$$\cos\theta+\sin\theta=\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right]$$

$$=\sqrt{2}\sin(\pi/4+\theta)$$

$\therefore \cos\theta+\sin\theta\le\sqrt{2}<\pi/2$ $\left(\because \begin{matrix}\sqrt{2}=1.414\\ \pi/2=1.57\end{matrix}\right)$

$\therefore \cos\theta+\sin\theta<\pi/2 \Rightarrow \cos\theta<\pi/2-\sin\theta$ ...(1)

As $\theta\in[0,\pi/2]$ in which $\sin\theta$ increases.

$\therefore$ Taking sin on both sides of eq. (1), we get

$\sin(\cos\theta)<\sin(\pi/2-\sin\theta)$

$\sin(\cos\theta)<\cos(\sin\theta)$

$\Rightarrow \cos(\sin\theta)>\sin(\cos\theta)$ ....(1)

Hence the result.

6. L.H.S. $=\sin 12°\sin 48°\sin 54°=\frac{1}{2}[2\sin 12°\cos 42°]\sin 54°$

$$=\frac{1}{2}\sin^2 54°-\frac{1}{2}\sin 54°=\frac{1}{4}\left[2\sin^2 54°-\sin 54°\right]$$

Now we know that $\sin 54°=\frac{1+\sqrt{5}}{4}$

$\therefore$ We get, $=\frac{1}{4}\left[2\left(\frac{1+\sqrt{5}}{4}\right)^2-\left(\frac{1+\sqrt{5}}{4}\right)\right]$

$$=\frac{1}{4}\left[2\left(\frac{1+5+2\sqrt{5}}{16}\right)-\left(\frac{1+\sqrt{5}}{4}\right)\right]$$

$$=\frac{1}{4}\times\frac{1}{8}\left[6+2\sqrt{5}-2-2\sqrt{5}\right]$$

$$=\frac{1}{32}\times 4=\frac{1}{8}=\text{R.H.S.}$$

7. We know that,

$$\cos A\cos 2A\cos 4A\ldots\cos 2^n A=\frac{1}{2^{n+1}\sin A}\sin.(2^{n+1}A)$$

$\therefore 16\cos\frac{2\pi}{15}\cos 2\left(\frac{2\pi}{15}\right)\cos 2^2\left(\frac{2\pi}{15}\right)\cos 2^3\left(\frac{2\pi}{15}\right)$

$=16.\frac{\sin(2^4 A)}{2^4\sin A}$ $\quad$ (where $A=2\pi/15$)

$$=16.\frac{\sin(32\pi/15)}{16\sin 2\pi/15}=\frac{\sin(32\pi/15)}{\sin(2\pi+2\pi/15)}=\frac{\sin(32\pi/15)}{\sin(32\pi/15)}=1$$

8. Given eq. is,

$4\cos^2 x\sin x-2\sin^2 x=3\sin x$

$\Rightarrow 4\cos^2 x\sin x-2\sin^2 x-3\sin x=0$

$\Rightarrow 4(1-\sin^2 x)\sin x-2\sin^2 x-3\sin x=0$

$\Rightarrow \sin x[4\sin^2 x+2\sin x-1]=0$

$\Rightarrow$ either $\sin x=0$ or $4\sin^2 x+2\sin x-1=0$

If $\sin x=0\Rightarrow x=n\pi$

$\Rightarrow$ If $4\sin^2 x+2\sin x-1=0\Rightarrow \sin x=\frac{-1\pm\sqrt{5}}{4}$

If $\sin x=\frac{-1\pm\sqrt{5}}{4}=\sin 18°=\sin\frac{\pi}{10}$

then $x=nx+(-1)^4\frac{\pi}{10}$

If $\sin x=-\left(\frac{\sqrt{5}+1}{4}\right)=\sin(-54°)=\sin\left(\frac{-3\pi}{10}\right)$

then $x=n\pi+(-1)^n\left(\frac{-3\pi}{10}\right)$

Hence, $x = n\pi,\ n\pi + (-1)^n \dfrac{\pi}{10}$ or $n\pi + (-1)^n\left(\dfrac{-3\pi}{10}\right)$

where n is some integer

9. The given equation is

$$8^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+....)} = 4^3$$

$\Rightarrow\ 2^{3(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+....)} = 2^6$

$\Rightarrow\ 3\,(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+...) = 6$

$\Rightarrow\ 1+|\cos x|+|\cos^2 x|+|\cos^3 x|+.... = 2$

$\Rightarrow\ \dfrac{1}{1-|\cos\ x|} = 2$ **NOTE THIS STEP**

$\Rightarrow\ 1-\cos x = 1/2 \Rightarrow |\cos x| = \dfrac{1}{2}$

$\Rightarrow\ x = \pi/3, -\pi/3, 2\pi/3, -2\pi/3, ...$

The values of $x \in (-\pi, \pi)$ are $\pm\ \pi/3,\ \pm\ 2\pi/3$.

10. We know that $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

$\Rightarrow\ \dfrac{1-\tan^2\alpha}{\tan\alpha} = 2\cot 2\alpha \Rightarrow \cot\alpha - \tan\alpha = 2\cot 2\alpha$

Now we have to prove

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

LHS

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 4\,(2\cot 2\,.\,4\alpha)$

$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 4\,(\cot 4\alpha - \tan 4\alpha)$ [Using (1)]

$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 4\cot 4\alpha - 4\tan 4\alpha$

$= \tan\alpha + 2\tan 2\alpha + 2\,(2\cot 2.\,2\ \alpha)$

$= \tan\alpha + 2\tan 2\alpha + 2\,(\cot\alpha - \tan 2\alpha)$

$= \tan\alpha + 2\tan 2\alpha + 2\,(2\cot 2\alpha - \tan 2\alpha)$ [Using (1)]

$= \tan\alpha + 2\cot 2\alpha$

$= \tan\alpha + (\cot\alpha - \tan\alpha)$ [Using (1)]

$= \cot\alpha =$ RHS.

11. Given that in ΔABC, A, B and C are in A.P.

$\therefore\ A + C = 2B$

also $A + B + C = 180° \Rightarrow B + 2B = 180° \Rightarrow B = 60°$

Also given that, $\sin(2A+B) = \sin(C-A) = -\sin(B+2C) = \dfrac{1}{2}$

$\Rightarrow\ \sin(2A+60°) = \sin(C-A) = -\sin(60+2C) = \dfrac{1}{2}$ ...(1)

From eq. (1), we have

$\sin(2A+60°) = \dfrac{1}{2} \Rightarrow 2A + 60° = 30°,\ 150°$

but A can not be –ve

$\therefore\quad 2A + 60° = 150° \Rightarrow 2A = 90° \Rightarrow A = 45°$

Again from (1) $\sin(60° + 2C) = -\dfrac{1}{2}$

$\Rightarrow\ 60° + 2C = 210°$ or $330°$

$\Rightarrow\ C = 75°$ or $135°$

Also from (1) $\sin(C-A) = \dfrac{1}{2} \Rightarrow C - A = 30°, 150°$

For $A = 45°$, $C = 75°$ or $195°$ (not possible) $\therefore\ C = 75°$

Hence we have $A = 45°$, $B = 60°$, $C = 75°$.

12. Let $y = \exp\,[\sin^2 x + \sin^4 x + \sin^6 x + ...\infty]\ \ln 2$

$= e^{ln\,\cdot\,2^{\sin^2 x+\sin^4 x+\sin^6 x+....\infty}}$

$= 2^{\sin^2 x+\sin^4 x+\sin^6 x+....\infty} = 2^{\frac{\sin^2 x}{1-\sin^2 x}} = 2^{\tan^2 x}$

As y satisfies the eq.

$x^2 - 9x + 8 = 0 \qquad \therefore\ y^2 - 9y + 8 = 0$

$\Rightarrow\ (y-1)\ (y-8) = 0 \Rightarrow y = 1, 8$

$\Rightarrow\ 2^{\tan^2 x} = 1$ or $2^{\tan^2 x} = 8$

$\Rightarrow\ \tan^2 x = 0$ or $\tan^2 x = 3$

$\Rightarrow \tan x = 0$ or $\tan x = \sqrt{3}, -\sqrt{3}$

$\Rightarrow\ x = 0$ or $x = \pi/3,\ 2\pi/3$

But given that $0 < x < \pi/2 \Rightarrow x = \pi/3$

Hence $\dfrac{\cos x}{\cos x + \sin x} = \dfrac{1}{1+\tan x} = \dfrac{1}{1+\sqrt{3}} = \dfrac{\sqrt{3}-1}{2}$

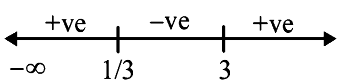
13. Let $y = \dfrac{\tan x}{\tan 3x} \Rightarrow y = \dfrac{\tan x(1-3\tan^2 x)}{3\tan x - \tan^3 x}$

$\Rightarrow\ 3y - 3\tan^2 x = 1 - 3\tan^2 x$

$\Rightarrow\ (y-3)\tan^2 x = 3y - 1 \Rightarrow \tan^2 x = \dfrac{3y-1}{y-3}$

$\Rightarrow\ \dfrac{3y-1}{y-3} > 0$ (L.H.S. being a prefect square)

$\Rightarrow\ \dfrac{(3y-1)\ (y-3)}{(y-3)^2} > 0 \Rightarrow (3y-1)\ (y-3) > 0$

$\Rightarrow$ 

$\Rightarrow\ y < \dfrac{1}{3}$ *or* $y > 3$

Thus y never lies between $\dfrac{1}{3}$ and 3.

14. Given that,

$\tan(x+100°) = \tan(x+50°)\tan x \tan(x-50°)$

$\Rightarrow\ \dfrac{\tan(x+100°)}{\tan x} = \tan(x+50°)\ \tan(x-50°)$

$\Rightarrow\ \dfrac{\sin(x+100°)\cos x}{\cos(x+100°)\sin x} = \dfrac{\sin(x+50°)\ \sin(x-50°)}{\cos(x+50°)\cos(x-50°)}$

$\Rightarrow\ \dfrac{\sin(2x+100°)+\sin 100°}{\sin(2x+100°)-\sin 100°} = \dfrac{\cos 100° - \cos 2x}{\cos 100° + \cos 2x}$

Applying componendo and dividendo, we get

$\Rightarrow \dfrac{2\sin(2x+100°)}{2\sin 100°} = \dfrac{2\cos 100°}{-2\cos 2x}$

$\Rightarrow$ 2 sin (2x + 100°) cos 2x = – 2 sin 100° cos 100°

$\Rightarrow$ sin (4x + 100°) + sin 100° = – sin 200°

$\Rightarrow$ sin (4x + 10° + 90°) + sin (90° + 10°) = – sin (180 + 20°)

$\Rightarrow$ cos (4x + 10°) + cos 10° = sin 20°

$\Rightarrow$ cos (4x + 10°) = sin 20° – cos 10°

$\Rightarrow$ cos (4x + 10°) = sin 20° – sin 80°

$= -2\cos 50°\sin 30° = -2\cos 50°.\dfrac{1}{2} = -\cos 50° = \cos 130°$

$\Rightarrow$ 4x + 10° = 130° $\Rightarrow$ x = 30°

15. Given that $\cos\theta = \sin\phi$

where $\theta = p\sin x,\ \phi = p\cos x$

Above is possible when both $\theta = \phi = \dfrac{\pi}{4}$ or $\theta = \phi = \dfrac{5\pi}{4}$

$\therefore\ p\sin x = \dfrac{\pi}{4}$ or $p\sin x = \dfrac{5\pi}{4}$

and $p\cos x = \dfrac{\pi}{4}$ or $p\cos x = \dfrac{5\pi}{4}$

Squaring and adding, $p^2 = \dfrac{\pi^2}{16}.2$ or $\dfrac{25\pi^2}{16}.2$

$\therefore\ p = \dfrac{\pi}{4}\sqrt{2}$ only for least positive value or $p = \dfrac{\pi}{4}\sqrt{2}$

16. **Given :** $(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$

or $(1-\tan^2\theta)(1+\tan^2\theta) + 2^{\tan^2\theta} = 0$

Let us put $\tan^2\theta = t$

$\therefore\ (1-t)(1+t)+2^t = 0$ or $1-t^2+2^t = 0$

It is clearly satisfied by $t = 3$.

as $-8+8=0\ \therefore\ \tan^2\theta = 3$

$\therefore\ p = \pm\pi/3$ in the given interval.

17. Let $y = \dfrac{\sin x\cos 3x}{\sin 3x\cos x} = \dfrac{\tan x}{\tan 3x}$

We have $y = \dfrac{\tan x}{\tan 3x} = \dfrac{\tan x(1-3\tan^2 x)}{3\tan x - \tan^3 x} = \dfrac{1-3\tan^2 x}{3-\tan^2 x}$

(the expression is not defined if $\tan x = 0$)

$\Rightarrow 3y - (\tan^2 x)y = 1 - 3\tan^2 x \Rightarrow 3y - 1 = (y-3)\tan^2 x$

$\Rightarrow \tan^2 x = \dfrac{3y-1}{y-3} = \dfrac{(3y-1)(y-3)}{(y-3)^2}$

Since $\tan^2 x > 0$, we get $(3y-1)(y-3) > 0$

$\Rightarrow \left(y - \dfrac{1}{3}\right)(y-3) > 0 \Rightarrow y < \dfrac{1}{3}$ or $y > 3$

This shows that y cannot lie between $\dfrac{1}{3}$ and 3.

18. Expanding the sigma on putting $k = 1, 2, 3, ..., n$

$S = (n-1)\cos\dfrac{2\pi}{n} + (n-2)\cos 2.\dfrac{2\pi}{n} + ...$

$+ 1.\cos(n-1)\cos\dfrac{2\pi}{n}$ ...(1)

We know that $\cos\theta = \cos(2\pi - \theta)$

Replacing each angle θ by $2\pi - \theta$ in (1), we get

$S = (n-1)\cos(n-1)\dfrac{2\pi}{n} + (n-2)\cos(n-2)\dfrac{2\pi}{n} + ...$

$+ 1.\cos\dfrac{2\pi}{n}$ by (1) .....(2)

Add terms in (1) and (2) having the same angle and take n common

$\therefore 2S = n\left[\cos\dfrac{2\pi}{n} + \cos\dfrac{4\pi}{n} + \cos\dfrac{6\pi}{n} + ... + \cos(n-1)\dfrac{2\pi}{n}\right]$

Angles are in A.P. of $d = \dfrac{2\pi}{n}$

$2S = n\left[\dfrac{\sin(n-1)\dfrac{\pi}{n}}{\sin\dfrac{\pi}{n}}\cos\dfrac{\dfrac{2\pi}{n}+(n-1)\dfrac{2\pi}{n}}{2}\right]$ **NOTE THIS STEP**

$= n.1\cos\pi = -n \because \sin(\pi-\theta) = \sin\theta\ \therefore\ S = -n/2$

19. We have, $A + B + C = \pi$

$\Rightarrow \dfrac{A}{2}+\dfrac{B}{2}+\dfrac{C}{2} = \dfrac{\pi}{2} \Rightarrow \dfrac{A}{2}+\dfrac{B}{2} = \dfrac{\pi}{2} - \dfrac{C}{2}$

or $\cot\left(\dfrac{A}{2}+\dfrac{B}{2}\right) = \cot\left(\dfrac{\pi}{2}-\dfrac{C}{2}\right)$

$\Rightarrow \dfrac{\cot\dfrac{A}{2}.\cot\dfrac{B}{2} - 1}{\cot\dfrac{A}{2} + \cot\dfrac{B}{2}} = \tan\dfrac{C}{2}$

$\Rightarrow \cot\dfrac{A}{2} + \cot\dfrac{B}{2} + \cot\dfrac{C}{2} = \cot\dfrac{A}{2}.\cot\dfrac{B}{2}.\cot\dfrac{C}{2}$

20. Given that, $2\sin t = \dfrac{1-2x+5x^2}{3x^2-2x-1}$, $t \in [-\pi/2, \pi/2]$

This can be written as

$(6\sin t - 5)x^2 + 2(1-2\sin t)x - (1+2\sin t) = 0$

For given equation to hold, x should be some real number, therefore above equation should have real roots i.e., $D \geq 0$

$\Rightarrow 4(1-2\sin t)^2 + 4(6\sin t - 5)(1+2\sin t) \geq 0$

$\Rightarrow 16\sin^2 t - 8\sin t - 4 \geq 0 \Rightarrow (4\sin^2 t - 2\sin t - 1) \geq 0$

$\Rightarrow 4\left(\sin t - \dfrac{\sqrt{5}+1}{4}\right)\left(\sin t + \dfrac{\sqrt{5}-1}{4}\right) \geq 0$

$\Rightarrow \quad \sin t \le -\left(\frac{\sqrt{5}-1}{4}\right)$ or $\sin t \ge \frac{\sqrt{5}+1}{4}$

$\Rightarrow \quad \sin t \le \sin(-\pi/10)$ or $\sin t \ge \sin(3\pi/10)$

$\Rightarrow \quad t \le -\pi/10$ or $t \ge 3\pi/10$

(Note that $\sin x$ is an increasing function from $-\pi/2$ to $\pi/2$)

$\therefore$ range of t is $[-\pi/2, -\pi/10] \cup [3\pi/10, \pi/2]$.

F. Match the Following

1. (i) If $\frac{13\pi}{48} < \alpha < \frac{14\pi}{48} \Rightarrow \frac{13\pi}{16} < 3\alpha < \frac{14\pi}{16}$

and $\frac{13\pi}{24} < 2\alpha < \frac{14\pi}{24}$

$\Rightarrow 3\alpha \in II$ quad and $2\alpha \in II$ quad $\Rightarrow \sin 3\alpha = +$ ve

$\cos 2\alpha = -$ ve $\quad \therefore \frac{\sin 3\alpha}{\cos 2\alpha} = -ve$

$\therefore$ (B) corresponds to (p).

If $\alpha \in \left(\frac{14\pi}{48}, \frac{18\pi}{48}\right) \Rightarrow \frac{14\pi}{16} < 3\alpha < \frac{18\pi}{16}$

and $\frac{14\pi}{24} < 2\alpha < \frac{18\pi}{24}$

$\Rightarrow 3\alpha \in II$ or III quad and $2\alpha \in II$ quad

$\Rightarrow$ Nothing can be said about the sign of $\frac{\sin 3\alpha}{\cos 2\alpha}$ over this interval.

If $\alpha \in \left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$ then $\frac{18\pi}{16} < 3\alpha < \frac{23\pi}{16}$

and $\frac{18\pi}{24} < 2\alpha < \frac{23\pi}{24}$

$\Rightarrow 3\alpha \in III$ quad and $2\alpha \in II$ quad

$\Rightarrow \sin 3\alpha = -$ ve, $\cos 2a = -$ ve $\quad \therefore \frac{\sin 3\alpha}{\cos 2\alpha} = +ve$

$\therefore$ (A) corresponds to (r)

If $\alpha \in (0, \pi/2)$

$\Rightarrow \quad 0 < 3\alpha < 3\pi/2$ and $2 < 2\alpha < \pi$

$\Rightarrow$ Nothing can be said about the sign of $\frac{\sin 3\alpha}{\cos 2\alpha}$ over the given interval.

I. Integer Value Correct Type

1. **(3)** The given equations are

$xyz \sin 3\theta = (y+z)\cos 3\theta \quad$ —(1)

$xyz \sin 3\theta = 2z \cos 3\theta + 2y \sin 3\theta \quad$ —(2)

$xyz \sin 3\theta = (y+2z)\cos 3\theta + y \sin 3\theta \quad$ —(3)

Operating (1) – (2) and (3) – (1), we get

$(\cos 3\theta - 2\sin\theta)y - (\cos 3\theta)z = 0$

and $\sin 3\theta\, y + (\cos 3\theta)z = 0$

which is homogeneous system of linear equation. But $y \ne 0, z \ne 0$

$\therefore \frac{\cos 3\theta - 2\sin 3\theta}{\sin 3\theta} = -\frac{\cos 3\theta}{\cos 3\theta} \Rightarrow \cos 3\theta = \sin 3\theta$

$\Rightarrow \tan 3\theta = 1 \Rightarrow 3\theta = n\pi + \frac{\pi}{4} \Rightarrow \theta = (4n+1)\frac{\pi}{12}, n \in Z$

For $\theta \in (0, \pi) \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

$\therefore$ Three such solutions are possible.

2. **(3)** $\tan\theta = \cot 5\theta, \theta \ne \frac{n\pi}{5}$

$\Rightarrow \cos\theta\cos 5\theta - \sin 5\theta \sin\theta = 0 \Rightarrow \cos 6\theta = 0$

$\Rightarrow 6\theta = \frac{-5\pi}{2}, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow \theta = \frac{-5\pi}{12}, \frac{-\pi}{4}, \frac{-\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$

Again $\sin 2\theta = \cos 4\theta = 1 - 2\sin^2 2\theta$

$\Rightarrow 2\sin^2 2\theta + \sin 2\theta - 1 = 0 \Rightarrow \sin 2\theta = -1, \frac{1}{2}$

$\Rightarrow 2\theta = \frac{-\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{-\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

So common solutions are $\theta = \frac{-\pi}{4}, \frac{\pi}{12}$ and $\frac{5\pi}{12}$

$\therefore$ Number of solutions = 3.

3. **(2)** Let $f(\theta) = \frac{1}{g(\theta)}$

where $g(\theta) = \sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta$

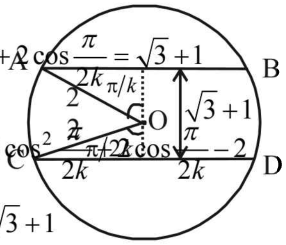
Clearly f is maximum when g is minimum

Now $g(\theta) = \frac{1-\cos 2\theta}{2} + \frac{3}{2}\sin 2\theta + \frac{5}{2}(1+\cos 2\theta)$

$= 3 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta \ge 3 + \left(-\sqrt{4+\frac{9}{4}}\right)$

$\therefore g_{min} = 3 - \frac{5}{2} = \frac{1}{2} \Rightarrow f_{max} = 2.$

4. **(3)** From figure, we get



$2\cos\frac{\pi}{k}+2\cos\frac{\pi}{2k}=\sqrt{3}+1$

$\Rightarrow 2\times 2\cos^2\frac{\pi}{2k}-2+2\cos\frac{\pi}{2k}$

$=\sqrt{3}+1$

$\Rightarrow 4\cos^2\frac{\pi}{2k}+2\cos\frac{\pi}{2k}-\left(3+\sqrt{3}\right)=0$

$\Rightarrow \cos\frac{\pi}{2k}=\frac{-2\pm\sqrt{4+16\left(3+\sqrt{3}\right)}}{8}=\frac{-1\pm\sqrt{13+4\sqrt{3}}}{4}$

$=\frac{-1\pm\left(2\sqrt{3}+1\right)}{4}=\frac{\sqrt{3}}{2}$ or $-\left(\frac{\sqrt{3}+1}{2}\right)$

As $\frac{\pi}{2k}$ is an acute angle, $\cos\frac{\pi}{2k}=\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6} \Rightarrow k=3$

5. **(7)** We have, $\frac{1}{\sin\frac{\pi}{n}}-\frac{1}{\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$

$\Rightarrow \frac{\sin\frac{3\pi}{n}-\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}} \Rightarrow \frac{2\cos\frac{2\pi}{n}\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$

$\Rightarrow 2\sin\frac{2\pi}{n}\cos\frac{2\pi}{n}=\sin\frac{3\pi}{n} \Rightarrow \sin\frac{4\pi}{n}-\sin\frac{3\pi}{n}=0$

$\Rightarrow 2\cos\frac{7\pi}{2n}\sin\frac{\pi}{2n}=0 \Rightarrow \cos\frac{7\pi}{2n}=0$ or $\sin\frac{\pi}{2n}=0$

$\Rightarrow \frac{7\pi}{2n}=(2k+1)\frac{\pi}{2}$ or $\frac{\pi}{2n}=2k\pi$ where $k\in Z$

$\Rightarrow n=\frac{7}{2k+1}$ or $n=\frac{1}{4k}$

($n=\frac{1}{4k}$ not possible for any integral value of k)

As $n>3$, for $k=0$, we get $n=7$.

6. **(8)** $\frac{5}{4}\cos^2 2x+\cos^4 x+\sin^4 x+\cos^6 x+\sin^6 x=2$

$\Rightarrow \frac{5}{4}\cos^2 2x+1-\frac{1}{2}\sin^2 2x+1-\frac{3}{4}\sin^2 2x=2$

$\Rightarrow \frac{5}{4}(\cos^2 2x-\sin^2 2x)=0 \Rightarrow \cos 4x=0$

$\Rightarrow 4x=(2n+1)\frac{\pi}{2}$ or $x=(2n+1)\frac{\pi}{8}$

For $x\in[0, 2\pi]$, n can take values 0 to 7

$\therefore$ 8 solutions.

Section-B JEE Main/ AIEEE

1. **(b)** $\sin^2\theta=\frac{1-\cos 2\theta}{2}$; Period $=\frac{2\pi}{2}=\pi$

2. **(b)** The given equation is $\tan x+\sec x=2\cos x$;

$\Rightarrow \sin x+1=2\cos^2 x \Rightarrow \sin x+1=2(1-\sin^2 x)$;

$\Rightarrow 2\sin^2 x+\sin x-1=0$;

$\Rightarrow (2\sin x-1)(\sin x+1)=0 \Rightarrow \sin x=\frac{1}{2},-1.$;

$\Rightarrow x=30°, 150°, 270°.$

3. **(b)** $\because \cos\sqrt{x}$ is non periodic

$\therefore \cos\sqrt{x}+\cos^2 x$ can not be periodic.

4. **(d)** $\pi<\alpha-\beta<3\pi$

$\Rightarrow \frac{\pi}{2}<\frac{\alpha-\beta}{2}<\frac{3\pi}{2} \Rightarrow \cos\frac{\alpha-\beta}{2}<0$

$\sin\alpha+\sin\beta=-\frac{21}{65}$

$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=-\frac{21}{65}$ .....(1)

$\cos\alpha+\cos\beta=-\frac{27}{65}$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=-\frac{27}{65}$ .....(2)

Square and add (1) and (2)

$4\cos^2\frac{\alpha-\beta}{2}=\frac{(21)^2+(27)^2}{(65)^2}=\frac{1170}{65\times 65}$

$\therefore \cos^2\frac{\alpha-\beta}{2}=\frac{9}{130} \Rightarrow \cos\frac{\alpha-\beta}{2}=-\frac{3}{\sqrt{130}}$

5. **(a)** $u^2=a^2+b^2+2\sqrt{\begin{matrix}(a^4+b^4)\cos^2\theta\sin^2\theta \\ +a^2b^2(\cos^4\theta+\sin^4\theta)\end{matrix}}$ ...(1)

Now $(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(\cos^4\theta+\sin^4\theta)$

$=(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(1-2\cos^2\theta\sin^2\theta)$

$=(a^4+b^4-2a^2b^2)\cos^2\theta\sin^2\theta+a^2b^2$

$=(a^2-b^2)^2.\dfrac{\sin^2 2\theta}{4}+a^2b^2$ ...(2)

$\because 0\le\sin^2 2\theta\le 1$

$\Rightarrow 0\le(a^2-b^2)^2\dfrac{\sin^2 2\theta}{4}\le\dfrac{(a^2-b^2)^2}{4}$

$\Rightarrow a^2b^2\le(a^2-b^2)^2\dfrac{\sin^2 2\theta}{4}+a^2b^2$

$\le(a^2-b^2)^2.\dfrac{1}{4}+a^2b^2$ ....(3)

$\therefore$ from (1), (2) and (3)

Minimum value of $u^2=a^2+b^2+2\sqrt{a^2b^2}=(a+b)^2$

Maximum value of u^2

$=a^2+b^2+2\sqrt{\left(a^2-b^2\right)^2.\dfrac{1}{4}+a^2b^2}$

$=a^2+b^2+\dfrac{2}{2}\sqrt{(a^2+b^2)^2}=2(a^2+b^2)$

$\therefore$ Max value - Min value

$=2(a^2+b^2)-(a+b^2)=(a-b)^2$

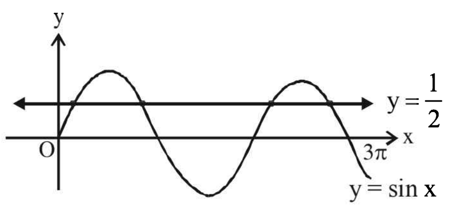
6. **(c)** The direction cosines of the line are $\cos\theta$, $\cos\beta$, $\cos\theta$

$\therefore \cos^2\theta+\cos^2\beta+\cos^2\theta=1$

$\Rightarrow 2\cos^2\theta=\sin^2\beta=3\sin^2\theta$ (given)

$\Rightarrow 2\cos^2\theta=3-3\cos^2\theta$ $\therefore \cos^2\theta=\dfrac{3}{5}$

7. **(a)**



$2\sin^2 x+5\sin x-3=0$

$\Rightarrow (\sin x+3)(2\sin x-1)=0$

$\Rightarrow \sin x=\dfrac{1}{2}$ and $\sin x\ne-3$

$\therefore$ In $[0,3\pi]$, x has 4 values.

8. **(c)** $\cos x+\sin x=\dfrac{1}{2}\Rightarrow 1+\sin 2x=\dfrac{1}{4}$

$\Rightarrow \sin 2x=-\dfrac{3}{4}$, so x is obtuse and

$\dfrac{2\tan x}{1+\tan^2 x}=-\dfrac{3}{4}\Rightarrow 3\tan^2 x+8\tan x+3=0$

$\therefore \tan x=\dfrac{-8\pm\sqrt{64-36}}{6}=-\dfrac{-4\pm\sqrt{7}}{3}$

as $\tan x<0$ $\therefore \tan x=\dfrac{-4-\sqrt{7}}{3}$

9. **(b)** We have

$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\dfrac{3}{2}$

$\Rightarrow 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+3=0$

$\Rightarrow 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]$
$+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta+\sin^2\gamma+\cos^2\gamma=0$

$\Rightarrow [\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\sin\beta+2\sin\beta\sin\gamma$
$+2\sin\gamma\sin\alpha]+[\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2\cos\alpha\cos\beta$
$+2\cos\beta\cos\gamma+2\cos\gamma\cos\alpha]=0$

$\Rightarrow [\sin\alpha+\sin\beta+\sin\gamma]^2+(\cos\alpha+\cos\beta+\cos\gamma)^2=0$

$\Rightarrow \sin\alpha+\sin\beta+\sin\gamma=0$ and $\cos\alpha+\cos\beta+\cos\gamma=0$

$\therefore$ A and B both are true.

10. **(a)** $\cos(\alpha+\beta)=\dfrac{4}{5}\Rightarrow\tan(\alpha+\beta)=\dfrac{3}{4}$

$\sin(\alpha-\beta)=\dfrac{5}{13}\Rightarrow\tan(\alpha-\beta)=\dfrac{5}{12}$

$\tan 2\alpha=\tan[(\alpha+\beta)+(\alpha-\beta)]=\dfrac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}}=\dfrac{56}{33}$

11. **(d)** $A=\sin^2 x+\cos^4 x=\sin^2 x+\cos^2 x(1-\sin^2 x)$

$=\sin^2 x+\cos^2 x-\dfrac{1}{4}(2\sin x.\cos x)^2=1-\dfrac{1}{4}\sin^2(2x)$

Now $0\le\sin^2(2x)\le 1\Rightarrow 0\ge-\dfrac{1}{4}\sin^2(2x)\ge-\dfrac{1}{4}$

$\Rightarrow 1\ge 1-\dfrac{1}{4}\sin^2(2x)\ge 1-\dfrac{1}{4}\Rightarrow 1\ge A\ge\dfrac{3}{4}$

12. **(b)** Given $3\sin P+4\cos Q=6$ ...(i)

$4\sin Q+3\cos P=1$ ...(ii)

Squaring and adding (i) & (ii) we get

$9\sin^2 P+16\cos^2 Q+24\sin P\cos Q$
$+16\sin^2 Q+9\cos^2 P+24\sin Q\cos P=36+1=37$

$\Rightarrow$ $9(\sin^2 P+\cos^2 P)+16(\sin^2 Q+\cos^2 Q)$
$+24(\sin P\cos Q+\cos P\sin Q)=37$

$\Rightarrow$ $9+16+24\sin(P+Q)=37$

[$\because \sin^2\theta+\cos^2\theta=1$ and $\sin A\cos B+\cos A\sin B = \sin(A+B)$]

$\Rightarrow$ $\sin(P+Q)=\frac{1}{2}$ $\Rightarrow$ $P+Q=\frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\Rightarrow$ $R=\frac{5\pi}{6}$ or $\frac{\pi}{6}$ $(\because P+Q+R=\pi)$

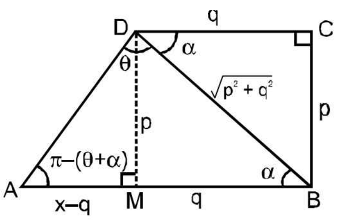
If $R=\frac{5\pi}{6}$ then $0<P,Q<\frac{\pi}{6}$

$\Rightarrow$ $\cos Q<1$ and $\sin P<\frac{1}{2}$

$\Rightarrow$ $3\sin P+4\cos Q<\frac{11}{2}$ which is not true.

So $R=\frac{\pi}{6}$

13. (a) From Sine Rule



$$\frac{AB}{\sin\theta}=\frac{\sqrt{p^2+q^2}}{\sin(\pi-(\theta+\alpha))}$$

$$AB=\frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha}=\frac{(p^2+q^2)\sin\theta}{q\sin\theta+p\cos\theta}$$

$$\left(\because \cos\alpha=\frac{q}{\sqrt{p^2+q^2}} \text{ and } \sin\alpha=\frac{p}{\sqrt{p^2+q^2}}\right)$$

14. (b) Given expression can be written as

$$\frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A}+\frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A}$$

$$\left(\because \tan A=\frac{\sin A}{\cos A} \text{ and } \cot A=\frac{\cos A}{\sin A}\right)$$

$$=\frac{1}{\sin A-\cos A}\left\{\frac{\sin^3 A-\cos^3 A}{\cos A\sin A}\right\}$$

$$=\frac{\sin^2 A+\sin A\cos A+\cos^2 A}{\sin A\cos A}=1+\sec A\operatorname{cosec} A$$

15. (b) Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$

Consider

$$f_4(x)-f_6(x)=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$$

$$=\frac{1}{4}[1-2\sin^2 x\cos^2 x]-\frac{1}{6}[1-3\sin^2 x\cos^2 x]$$

$$=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$$

16. (a) $\cos x+\cos 2x+\cos 3x+\cos 4x=0$

$\Rightarrow 2\cos 2x\cos x+2\cos 3x\cos x=0$

$$\Rightarrow 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right)=0$$

$$\cos x=0,\ \cos\frac{5x}{2}=0,\ \cos\frac{x}{2}=0$$

$$x=\pi,\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{5},\frac{3\pi}{5},\frac{7\pi}{5},\frac{9\pi}{5}$$